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Stochastic Decision-Support Modeling for Digital Supply Chain Management under Demand and Lead-Time Uncertainty

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Abstract

In digital supply chain management, the effectiveness of managerial decision-making depends on the ability of information systems to support timely and cost-efficient delivery planning under uncertainty. This study develops a stochastic decision-support model for product delivery planning under random consumer demand and variable delivery lead time. The proposed approach addresses the limitations of deterministic inventory models, which often fail to reflect the uncertainty of real logistics in information-management environments. The model minimizes expected total costs by balancing inventory holding costs and losses caused by stockouts. Deviations between scheduled and actual delivery time, as well as between expected and actual inventory depletion time, are represented as continuous normally distributed random variables. This enables the analytical derivation of the expected cost function and reduces the optimization problem to determining the optimal scheduled delivery time. The optimality condition is obtained through an integral equation and is shown to have a unique solution due to the monotonic behavior of the corresponding probability function. A numerical example and graphical analysis demonstrate how holding costs, shortage losses, and uncertainty levels affect delivery timing decisions. The results show that higher holding costs shift the optimal delivery time toward later deliveries, whereas higher shortage losses require earlier scheduling. Increased uncertainty strengthens the sensitivity of the decision-support model to planning errors. The proposed model can be used as an analytical component of digital inventory-control and supply chain management systems.

Keywords: IT management, Decision support model, Logistics, Sustainability of retail chains, Stochastic demand, Food security.

Introduction

Modern supply chains operate under conditions of uncertainty caused primarily by stochastic demand and variability in order lead time. These factors lead to deviations of actual product delivery times and the moment of inventory depletion from planned values, which complicates the coordination of actions among supply chain participants and increases the risk of shortages or excessive inventories. The relevance of developing this class of mathematical models is caused by the fact that existing deterministic approaches to planning deliveries and inventories often do not allow an adequate description of the real dynamics of markets and logistics processes, where demand and delivery times are subject to random fluctuations and delays.

As a result, enterprises face an increase in total costs and a decrease in the level of service. The application of stochastic models makes it possible to take into account the joint uncertainty of demand and lead time, assess the probability of shortages, and form decisions that ensure a balance between holding costs and losses from unmet demand. In addition, an analysis of existing studies shows that many models rely on simplifying assumptions (limited supply chain structure, single product, etc.), which makes their use for describing complex real supply chains difficult and therefore requires further development and refinement of stochastic formulations.

Literature Review

Research in this field is oriented both toward classical stochastic inventory management models and toward modern approaches that include dynamic optimization and the modeling of supply chain networks.

Debnath and Sarkar (2024) proposed an integrated “manufacturer - retailer” supply chain model for a single product, combining a variable production rate, demand dependent on price and advertising, and stochastic demand during lead time (LTD) under the SSMD shipment policy. Two variants for describing LTD are considered - assuming a normal distribution and in a distribution-free setting. It is shown that abandoning the distributional assumption strengthens the requirements for safety parameters, increasing the robustness of decisions at the cost of higher capital tied up in inventories, while the differences in final profit according to numerical experiments are insignificant. As noted by the authors themselves, the presented model is constructed for a simplified two-echelon structure ("one manufacturer - one retailer") and a single product; therefore, its results are only limitedly applicable to multi-echelon and multi-item supply chains. Kim and Sarkar (2017) extend classical formulations by simultaneously accounting for trade-credit financing, transportation discounts, as well as investments in reducing setup costs and improving product quality as coordination instruments in the supply chain (Kashchena et al., 2024; Kyrylieva et al., 2023; Nesterenko et al., 2024). The study aims to minimize total supply chain costs and to determine parameters that ensure a mutually beneficial outcome; an iterative algorithm is used to search for the optimum. Glock and Ries (2013) analyze multiple sourcing models with variable lead time and stochastic demand, while Alvarez et. al. (2021) consider the inventory routing problem using stochastic programming methods, in which both supply and demand are uncertain quantities.

Barman et. al. (2021) present a "supplier - buyer" model that accounts for stochastic demand during lead time and stochastic lead time, as well as financial conditions (advance payments). The inclusion of an environmental component expands the formulation to assess the trade-off between costs, service, and sustainability. Zhou et. al. (2012) investigate a "multiple suppliers - single buyer" system with «milkrun» deliveries, where lead time is stochastic and lead time variability can be reduced through additional "crashing" costs. The authors propose an integrated model that makes it possible to evaluate the trade-off between the costs of reducing delivery times and the effect of reducing uncertainty. Unlike classical formulations, Ben-Daya and Hariga (2004) specify delivery time as a variable and make it dependent on lot size, while Chang et. al. (2006) study a cooperative "supplier - buyer" model in which lead time may be reduced through additional "crashing" costs, and a reduction in order processing costs is also considered as a controllable factor. Onggo et. al. (2019) consider a problem in which demand is stochastic, whereas Modak and Kelle (2019) present a model that shows how joint decisions on prices, lot sizes, and delivery time affect profit and service level. Johansen (2019), as well as Riezebos and Zhu (2020), analyze periodic inventory review for a single product with stochastic lead times for "regular" orders and introduce a mechanism of emergency orders with a short constant lead time and higher cost (Babenko et al., 2019, 2020). It should be noted that the limitations of the presented models are related to simplified formulations: both models focus on a single item and periodic review, do not cover network supply chains, capacity constraints, and transportation structure, and also require the availability of stochastic lead time parameters for practical implementation.

It should be noted that despite the wide range of approaches proposed by the authors, most studies rely on simplifying assumptions (single item, two-echelon structure, limited transportation-network configuration, exogenous demand and lead time parameters, partial neglect of correlations and disruptions), which reduces the transferability of results to complex real-world supply chains. Therefore, despite the significant number of studies, further development is required in the direction of stochastic and multi-item models, accounting for the joint uncertainty of demand and lead time, integration of logistical constraints, and empirical calibration of robust replenishment strategies.

The purpose of the article is to develop a stochastic economic and mathematical model of product deliveries that makes it possible to determine the optimal scheduled delivery time under uncertainty in demand and order lead time based on the minimization of expected total costs. Within the framework of the study, a cost function including holding costs and shortage losses is formulated, and an analytical approach to its optimization is proposed. A separate objective is to investigate the impact of model parameters and the level of uncertainty on the optimal delivery time and the sensitivity of the obtained solution.

Methodology

In practice, uncertainty often arises due to inaccuracy or incompleteness of data on demand and supplies, time delays in the receipt of ordered goods, and other parameters of the logistics system. In this regard, accounting for and analyzing uncertain factors when constructing a mathematical model is fundamental. It is necessary to distinguish between two types of uncertainty: stochastic and complete.

The peculiarity of stochastic uncertainty is that it can be described by a random variable or a random process. Complete uncertainty, by contrast, is in principle not interpreted as a random object. Thus, the type of uncertainty determines the choice of mathematical models applied, and the subject of the study is specifically stochastic economic and mathematical models of inventory management systems.

As already noted, to solve problems with a priori uncertainty, modern inventory management literature proposes various approaches, including set-theoretic methods, scenario modeling, and simulation modeling. Figure 1 illustrates the features of inventory management models.

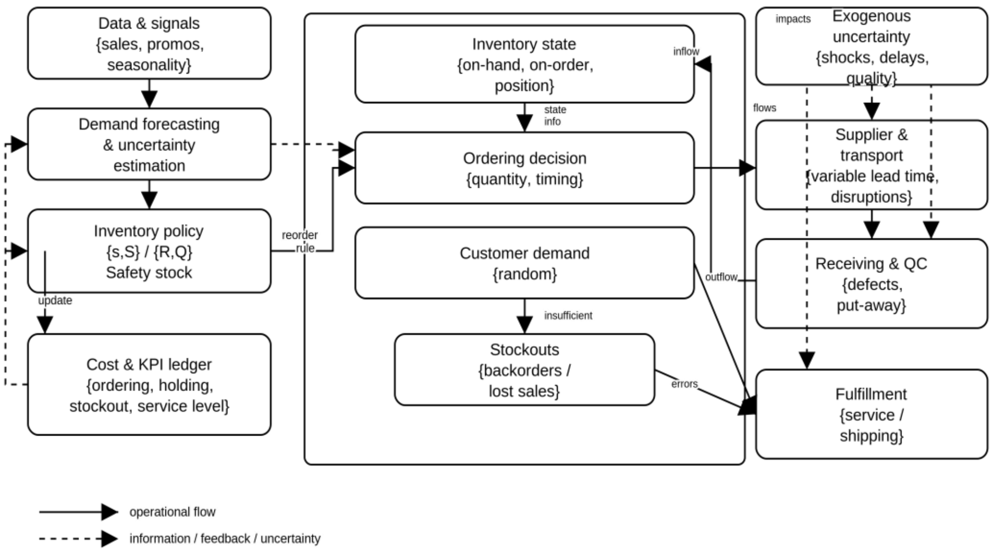

Figure 1. Stochastic Inventory Control System

The features of the analysis of inventory management models reflected in the constructed scheme are determined by a number of key factors. First, demand is considered a random process, which requires the use of forecasting and uncertainty assessment when forming replenishment policies. Second, the duration of supply procedures (including transportation, receipt, and quality control) is a random variable; therefore, decisions must take into account lead time variability and possible disruptions. Third, the problem arises of determining the replenishment quantity that ensures the required service level at acceptable holding and shortage costs. Finally, it is necessary to select order placement times and account for order arrival times, which in the scheme is represented by the relationship between the inventory state, the replenishment rule, and the supply loop. Taken together, these features justify the application of stochastic economic and mathematical models and the organization of feedback through a performance indicator block to adjust forecasting parameters and inventory management policies.

Assume that

$$\tau = \tilde{\tau} + \Delta\tau$$

is the time of the actual delivery of the product, where $\tilde{\tau}$ is the scheduled delivery time. Let

$$\delta = \delta_0 + \Delta\delta$$

is the time when the product is actually depleted in the warehouse, and $\delta_0$is the expected time of depletion. In the presented model, $\Delta\tau$ is a random variable describing the deviation of the actual delivery time from the scheduled delivery time, and δ is a random variable describing the deviation of the actual depletion time from the expected depletion time.

The cost function that reflects the effectiveness of the chosen inventory management strategy, within the framework of this study, includes holding costs and shortage costs. The latter are interpreted as lost profit and are assumed to be proportional to the time during which the required quantity of the product is unavailable in stock. Holding costs for the product in the assumed volume will be denoted by. V.

Holding costs for a batch of goods of a certain volume V after delivery over the time interval up to the moment of actual inventory depletion δ , in the case when the delivery arrives earlier than the expected time $(\tilde{\tau} + \Delta\tau < \delta)$, can be represented as follows:

$$Z = \gamma \cdot V \cdot (\delta - \tilde{\tau} - \Delta\tau) \quad (1)$$

where γ is a constant representing the daily holding cost of one unit of product.

In the case of incomplete demand satisfaction, and when the condition

$\tilde{\tau} + \Delta\tau > \delta,$

holds, shortage costs are incurred over the interval from the actual inventory depletion time δ to the delivery time τ for a volume V . These costs are determined by formula (2).

$$S = \frac{V}{\delta_0} \cdot \beta \cdot (\delta - \tilde{\tau} - \Delta\tau) \quad (2)$$

where β is the profit from selling one unit of product, and V/δ_0- represents the average daily volume of goods sold. Total costs can be determined by the following formula:

$$I + S = \begin{cases} \gamma \cdot V \cdot (\delta - \tilde{\tau} - \Delta\tau), \delta > \tilde{\tau} + \Delta\tau \\ \dfrac{V}{\delta_0} \cdot \beta \cdot (\delta + \Delta\tau - \delta), \tilde{\tau} + \Delta\tau > \delta \end{cases}$$

Results

Since the total costs in the stochastic formulation are random variables, the subsequent analysis is based on their mathematical expectation as the key characteristic of the average cost level. Within the adopted approximation, it is assumed that the uncertain parameters $\Delta\tau$ and δ are continuous random variables that follow a normal distribution with probability density functions $f_1(\Delta\tau)$ and $f_2(\Delta\delta)$ respectively. This assumption is used to obtain analytically convenient expressions for the expected costs.

$$K(\tilde{\tau}) = \int_{-\infty}^{+\infty} \left(\int_{-\infty}^{\delta-\tilde{\tau}} \gamma \cdot V \cdot (\delta - \tilde{\tau} - \Delta\tau) \cdot f_1(\Delta\tau) d\Delta\tau + \right. \\ \left. + \int_{\delta-\tilde{\tau}}^{+\infty} \frac{V}{\delta_0} \beta \cdot (\delta + \Delta\tau - \delta) \cdot f_1(\Delta\tau) d\Delta\tau \right) f_2(\Delta\delta) d(\Delta\delta) \tag{3}$$

The solution to the stated problem consists of finding such a value of $\tilde{\tau}$ at which the objective function attains its minimum.

$$K(\tilde{\tau}) \rightarrow min$$

To determine the minimum costs, we compute the derivative of the given function $K(\tilde{\tau})$ with respect to $\tilde{\tau}$

$$\frac{dK(\tilde{\tau})}{d\tilde{\tau}} = \int_{-\infty}^{+\infty} \left(\frac{\partial K_1(\tilde{\tau}, \delta)}{\partial \tilde{\tau}} + \frac{\partial K_2(\tilde{\tau}, \delta)}{\partial \tilde{\tau}} \right) \cdot f_2(\Delta\delta) d\Delta\delta \tag{4}$$

$$\frac{dK(\tilde{\tau})}{d\tilde{\tau}} = \int_{-\infty}^{+\infty} \left(-\gamma V \Phi \left(\frac{\delta - \tilde{\tau}}{\sigma_1} \right) + \frac{V}{\delta_0} \cdot \beta - \frac{V}{\delta_0} \cdot \beta \cdot \Phi \left(\frac{\delta - \tilde{\tau}}{\sigma_1} \right) \right) \cdot f_2(\Delta\delta) d\Delta\delta \tag{5}$$

where $\Phi(x)$ is the Laplace function.

Since $\delta = \delta_0 + \Delta\delta$, we obtain:

$$\frac{dK(\tilde{\tau})}{d\tilde{\tau}} = \int_{-\infty}^{\infty} \left(\left(-\gamma V - \frac{V}{\delta_0} \cdot \beta \right) \cdot \Phi \left(\frac{\delta_0 + \Delta\delta - \tilde{\tau}}{\sigma_1} \right) + + \frac{V}{\delta_0} \cdot \beta \right) \cdot f_2(\Delta\delta) d\Delta\delta \tag{6}$$

Setting the computed derivative equal to zero, we obtain:

$$\int_{-\infty}^{\infty} \left(\Phi \left(\frac{\delta_0 + \Delta\delta - \tilde{\tau}}{\sigma_1} \right) \right) \cdot f_2(\Delta\delta) d\Delta\delta = \frac{1}{\frac{\gamma\delta_0}{\beta} + 1} \tag{7}$$

In the resulting integral equation, the quantity on the right-hand side characterizes the economic balance between holding costs and shortage losses.

Let us denote

$$I(\tilde{\tau}) = \int_{-\infty}^{\infty} \left(\Phi \left(\frac{\delta_0 + \Delta\delta - \tilde{\tau}}{\sigma_1} \right) \right) \cdot f_2(\Delta\delta) d\Delta\delta$$

This relationship represents the probability that a delivery scheduled for time $\tilde{\tau}$, taking into account the random deviation $\Delta\tau$, will arrive before the actual inventory depletion time δ. Consequently, a shortage occurs under the condition

$$\tilde{\tau} + \Delta\tau > \delta_0 + \Delta\delta$$

and its probability is equal to

$$.1 - I(\tilde{\tau})$$

As $\tilde{\tau}$ increases, the event $\tilde{\tau} + \Delta\tau > \delta_0 + \Delta\delta$ becomes less likely, and $I(\tilde{\tau})$ decreases monotonically.

The quantity

$$\mathrm{R}(\delta_0) = \frac{1}{\left(\frac{\gamma\delta_0}{\beta} + 1\right)}$$

characterizes the probability of avoiding a shortage that should be ensured so that the marginal holding costs and the marginal shortage losses are balanced.

Within the developed stochastic model, the optimal time for scheduling the delivery $\tilde{\tau}$ is determined by the condition that the expected total cost is minimized. This condition leads to an integral equation of the form:

$$I(\tilde{\tau}) = R(\delta_0)$$

where the function

$$I(\tilde{\tau}) = P\{\tilde{\tau} + \Delta\tau \le \delta_0 + \Delta\delta\}$$

characterizes the probability that a delivery scheduled for time τ, taking into account a random deviation $\Delta\tau$, will arrive before the actual time when inventories are depleted. The quantity $R(\delta_0)$ specifies an economically justified acceptable level of stockout risk, determined by the relationship between the unit holding cost γ and the losses from a stockout β.

With fixed values of the parameters γ, β, and δ_0, the quantity $R(\delta_0)$ is a constant and does not depend on the delivery scheduling time. At the same time, the function $I(\tilde{\tau})$ is a monotonically decreasing function of $\tilde{\tau}$. Indeed, increasing the delivery scheduling time leads to an increase in the probability of the event

$$\tilde{\tau} + \Delta\tau > \delta_0 + \Delta\delta$$

that is, to an increase in the risk of a stockout and, consequently, to a decrease in the probability that the delivery arrives on time. Thus, as $\tilde{\tau}$ increases, the value of the function $I(\tilde{\tau})$ decreases monotonically.

From the monotonicity of $I(\tilde{\tau})$ and the constancy of the right-hand side of the equation, it follows that the optimality equation has a unique solution τ^*, corresponding to the intersection point of the graph of $I(\tilde{\tau})$ with the horizontal line $R(\delta_0)$. This solution is interpreted as the delivery scheduling time at which a balance is achieved between the marginal holding costs and the marginal losses from a stockout.

It should be noted that, in the general case, in the presence of discontinuities, degenerate distributions, or abnormal situations (for example, when the distribution functions are defined piecewise), the solution of the optimality equation may take the form of an interval of values. However, in the model considered here, under the assumption that the random variables $\Delta\tau$ and $\Delta\delta$ have continuous distributions (in particular, normal ones), the function $I(\tilde{\tau})$ is continuous and strictly monotonic, which guarantees the existence and uniqueness of the optimal solution.

To obtain an analytical expression for the probability that the delivery arrives on time, the paper assumes that the random deviations of the actual delivery time $\Delta\tau$ and the actual inventory depletion time $\Delta\delta$ are independent, normally distributed random variables.

Let the difference of the above random variables

$$H = (\delta_0 + \Delta\delta) - (\tilde{\tau} - \Delta\tau)$$

characterize the time buffer between the actual moment when inventories are depleted and the actual moment when the delivery arrives. The delivery is considered on time if the following condition holds:

$$H \geq 0$$

Since the difference of independent normally distributed random variables is also normally distributed, the random variable H follows a normal distribution with expected value:

$$E[H] = \delta_0 - \tilde{\tau}$$

and variance:

$$Var[H] = \sigma_\tau^2 + \sigma_\delta^2$$

Introducing the notation

$$\sigma = \sqrt{\sigma_\tau^2 + \sigma_\delta^2}$$

the probability of no stockout can be written in the following form:

$$I(\tilde{\tau}) = P(H \geq 0) = \Phi\left(\frac{\delta_0 - \tilde{\tau}}{\sigma}\right),$$

where $\Phi\left(\frac{\delta_0 - \tilde{\tau}}{\sigma}\right)$ is the cumulative distribution function of the standard normal distribution (the Laplace function).

The resulting expression has a number of important properties. First, the function $I(\tilde{\tau})$ is continuous and strictly monotonically decreasing in $\tilde{\tau}$, which reflects the increase in the risk of a stockout when the delivery is scheduled later. Second, the analytical form of $I(\tilde{\tau})$ makes it possible to explicitly relate the optimal delivery scheduling time to the uncertainty parameters of the logistics system.

Substituting the analytical expression for $I(\tilde{\tau})$ into the optimality condition

$$I(\tilde{\tau}) = R(\delta_0)$$

Makes it possible to obtain an explicit solution for the optimal delivery scheduling time:

$$\tau^* = \delta_0 - \sigma\Phi^{-1}\big(R(\delta_0)\big),$$

where Φ^{-1} is the inverse of the standard normal cumulative distribution function.

Thus, under the assumption of normally distributed random deviations, the optimization problem admits an analytical solution, which significantly simplifies the practical application of the model and the conduct of sensitivity analysis.

The optimal delivery scheduling time is determined by three groups of parameters: the expected inventory depletion time δ_0, the level of uncertainty σ, and the economic trade-off between holding costs and stockout losses, as reflected in the quantity $R(\delta_0)$.

In real logistics systems, deviations in the actual delivery time $\Delta\tau$ and in the actual inventory depletion time $\Delta\delta$ are not always independent. In practice, situations may arise in which an increase in demand is accompanied by congestion of transport and warehouse infrastructure, which simultaneously accelerates inventory depletion and increases delivery delays. Under such conditions, a statistical dependence emerges between the random variables $\Delta\tau$ and $\Delta\delta$, which should be taken into account in modeling.

Assuming that the correlation coefficient between the random variables $\Delta\tau$ and $\Delta\delta$ is equal to $-1 \leq \rho \leq 1$, the variance will be equal to

$$Var(H) = \sigma_\tau^2 + \sigma_\delta^2 - 2\rho\sigma_\tau\sigma_\delta$$

Denoting

$$\sigma_\rho = \sqrt{\sigma_\tau^2 + \sigma_\delta^2 - 2\rho\sigma_\tau\sigma_\delta}$$

We obtain a generalized analytical expression for the probability that the delivery arrives on time:

From the resulting formula, it follows that the presence of a positive correlation between $\rho > 0$ deviations in delivery time and inventory depletion time lead to a decrease in the effective variance σ_ρ^2 and, consequently, to a shift in the optimal delivery scheduling time.

Economically, this means that when rising demand and delivery delays occur simultaneously, the system becomes more sensitive to planning errors, and ignoring correlation may result in an underestimated stockout risk.

In the case of negative correlation $\rho < 0$, the variability of the difference between the two times increases, which calls for a more conservative inventory replenishment policy. Thus, accounting for the correlation between key stochastic factors improves the model's adequacy and expands its applicability for analyzing real supply chains.

To verify the derived analytical results and to test the correctness of the proposed model, graphical relationships were constructed (shown in Fig. 2 and Fig. 3) that illustrate the solution of the optimality equation and the behavior of the expected total cost function.

Figure 2 shows the function $I(\tilde{\tau})$, which characterizes the probability that a delivery arrives on time - i.e., before the actual moment when inventories are depleted - as well as a horizontal line corresponding to the value $R(\delta_0)$. The graph of $I(\tilde{\tau})$ is monotonically decreasing, which fully agrees with the analytical expression

$$I(\tilde{\tau}) = \Phi\left(\frac{\delta_0 - \tilde{\tau}}{\sigma}\right)$$

and with the economic interpretation of the model, postponing the delivery scheduling time increases the risk of a stockout. The intersection of the curve $I(\tilde{\tau})$ with the level $R(\delta_0)$ uniquely determines the optimal delivery scheduling time τ^*. The absence of additional intersection points confirms the theoretical conclusion about the uniqueness of the solution to equation (7).

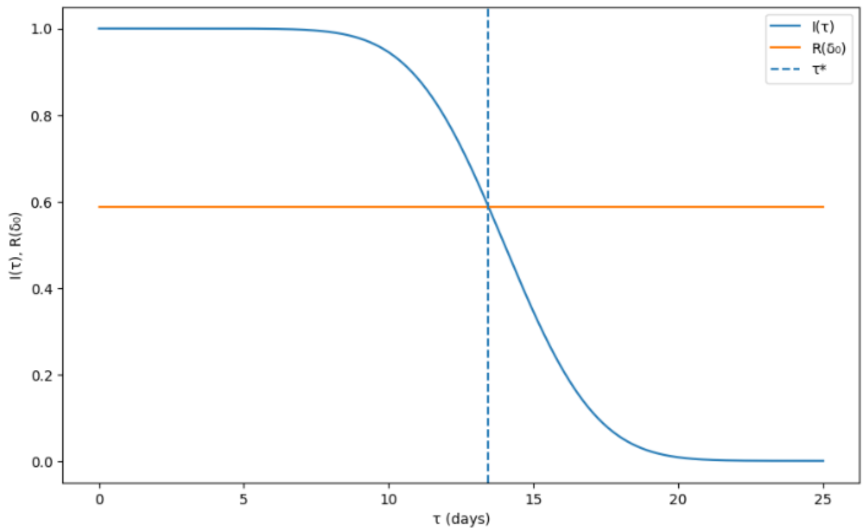

Figure 2. Numerical solution of equation (7)

Figure 3 illustrates the dependence of the expected total cost $K(\tilde{\tau})$ on the delivery scheduling time. The resulting curve has a convex, U-shaped form, which is characteristic of expected cost functions when they include opposing components - holding costs and stockout losses. The minimum of the function is attained at the same point τ^* that was determined from the condition $I(\tilde{\tau}) = R(\delta_0)$. This совпадение (coincidence) confirms the consistency of the analytical optimality condition with the results of numerical simulation.

It should be noted that as one moves away from the optimal value τ^*, the increase in expected costs is asymmetric. If the delivery is scheduled too early, the increase in costs is mainly due to higher holding costs, whereas if the delivery is scheduled too late, stockout losses dominate. Such asymmetry reflects the economic nature of the problem and further confirms the correctness of the chosen cost function.

In addition, an analysis of the shapes of the curves in Fig. 2 and Fig. 3 shows that as the level of uncertainty increases (i.e., as the variances σ_τ^2 and σ_δ^2, the graph of $I(\tilde{\tau})$ becomes flatter, and the minimum of the expected cost function becomes less pronounced. This indicates greater sensitivity of total costs to errors in choosing the delivery time and highlights the practical importance of accurately estimating the stochastic parameters of the model.

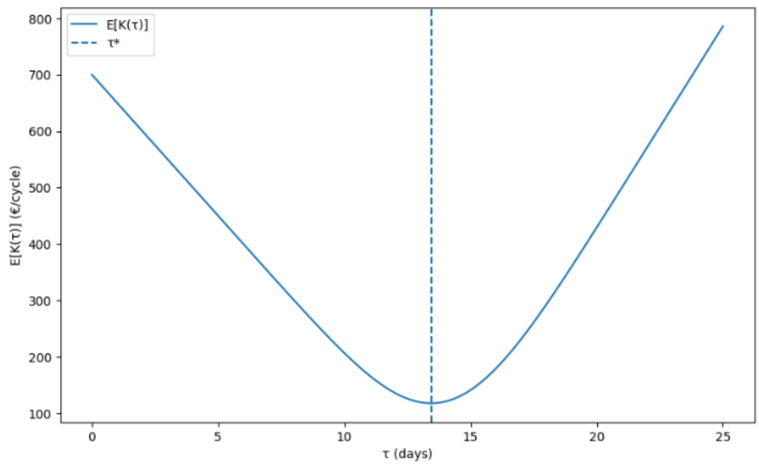

Figure 3. Graph of the dependence of expected costs on τ

Discussion

To test the applicability of the proposed stochastic model and to provide a clear illustration of the derived analytical results, a numerical calculation was performed using a set of parameters representing a typical inventory management scenario under uncertainty. The parameters were selected so that they have an economically interpretable meaning and, at the same time, make it possible to reproduce the values shown in Fig. 2 and Fig. 3.

The expected inventory depletion time was set to $\delta_0 = 14$ days, which corresponds to the average period required to sell the available stock of the product. The unit holding cost was specified as $\gamma = 0.05$ monetary units per unit of product per day, and the stockout loss as $\beta = 1$ monetary unit per unit of unsold product, corresponding to a situation in which lost profit substantially exceeds daily holding costs. The order quantity was taken as $V = 1000$ units.

Stochastic deviations in the delivery time and the inventory depletion time are modeled as normally distributed random variables with variances $\sigma_\tau^2 = 4$ and $\sigma_\delta^2 = 1$, reflecting higher variability in logistics delays compared to demand variability. In this case, the total variance of the difference between the two times is

$$\sigma = \sqrt{\sigma_\tau^2 + \sigma_\delta^2} = 2.24$$

With the specified parameters, the value

$$R(\delta_0) \approx 0.59$$

sets the target probability of avoiding a stockout, at which a balance is achieved between marginal holding costs and marginal stockout losses.

Substituting this value into the analytical expression for the optimal delivery scheduling time yields $\tau^* = 13.5$ days, which is fully consistent with the results shown in Fig. 2 and Fig. 3.

Thus, the optimal strategy is to schedule the delivery slightly earlier than the expected inventory depletion time, which compensates for the stochastic uncertainty of logistics processes.

In addition, the calculation shows that the probability of a stockout at the optimum is

$$1 - R(\delta_0) = 0.41$$

that is, under the given parameters, it is economically justified to accept a relatively high stockout risk because the holding losses associated with scheduling the delivery earlier would exceed the expected losses from an undersupply.

A numerical analysis of the expected total cost function confirms that, in the neighborhood of the optimal value τ^*, the function has a clearly pronounced minimum, and any deviation from the optimum in either direction leads to higher costs due to the dominance of either holding costs (for earlier deliveries) or stockout losses (for later deliveries). This confirms the robustness of the solution found and its consistency with the economic logic of the model.

Thus, the illustrative calculation demonstrates the possibility of practical use of the proposed model and shows that the derived analytical expressions make it possible not only to qualitatively but also quantitatively evaluate optimal decisions in inventory management problems under uncertainty.

To assess the robustness of the obtained optimal solution and to determine how the key model parameters affect the optimal delivery scheduling time τ^*, a sensitivity analysis was carried out with respect to the main economic and stochastic parameters: the unit holding cost γ, the stockout loss β, and the level of uncertainty characterized by the overall standard deviation σ.

Table 1 presents the results of calculating the optimal delivery time while varying one parameter and keeping the others fixed ($\delta_0 = 14$ days, $\gamma = 0.05, \beta = 1, V = 1000$).

Table 1. Sensitivity of the optimal delivery time to the model parameters

Parameter	Parameter value	$R(\delta_0)$	Optimal τ* (days)	Interpretation
(holding cost)γ	0.03	0.70	12.8	High shortage impact on earlier delivery
(holding cost)γ	0.05	0.59	13.5	Baseline case
(holding cost)γ	0.08	0.47	14.3	High holding cost for later delivery
β (shortage loss)	0.7	0.50	14.0	Lower shortage losses
β (shortage loss)	1.0	0.59	13.5	Baseline case
β (shortage loss)	1.5	0.68	12.9	Shortage critical for earlier delivery
(uncertainty)σ	1.5	0.59	13.8	Low uncertainty
(uncertainty)σ	2.24	0.59	13.5	Baseline case
(uncertainty)σ	3.0	0.59	13.0	High uncertainty

Source: compiled by the authors

The analysis shows that the optimal delivery scheduling time is most sensitive to the trade-off between stockout losses and holding costs, reflected in the quantity $R(\delta_0)$. An increase in the unit holding cost γ reduces the target probability of avoiding a stockout and shifts the optimal delivery time toward later values. Conversely, an increase in the stockout loss β makes earlier deliveries preferable despite the additional holding costs.

The level of stochastic uncertainty has a milder but systematic effect on the optimal decision. As the overall variance increases, the optimal delivery scheduling time shifts toward earlier values; however, the main impact is an increase in the sensitivity of total costs to planning errors. This means that under high uncertainty, even small deviations from the optimal delivery time can lead to a substantial rise in expected costs.

These results confirm that the proposed model can be used not only to determine the optimal delivery time, but also as an analytical tool for assessing the consequences of changes in economic conditions and in the level of uncertainty in supply chains.

Figures 4a - 4c show how the optimal delivery scheduling time τ^* depends on the main model parameters, while the remaining parameters are fixed at their baseline values. The baseline parameter values and the corresponding optimal value τ^* are marked with dashed lines.

Figure 4a indicates that increasing the unit holding cost γ shifts the optimal delivery time toward later dates, since earlier arrival of the batch entails higher holding costs. Conversely, an increase in the stockout loss β (Fig. 4b) leads to earlier scheduling of the delivery, reflecting the desire to reduce the risk of unmet demand.

Figure 4b shows that a higher level of uncertainty σ also shifts the optimal delivery time toward earlier values. This is due to the need to build in an additional time buffer when delivery lead times and demand exhibit high variability.

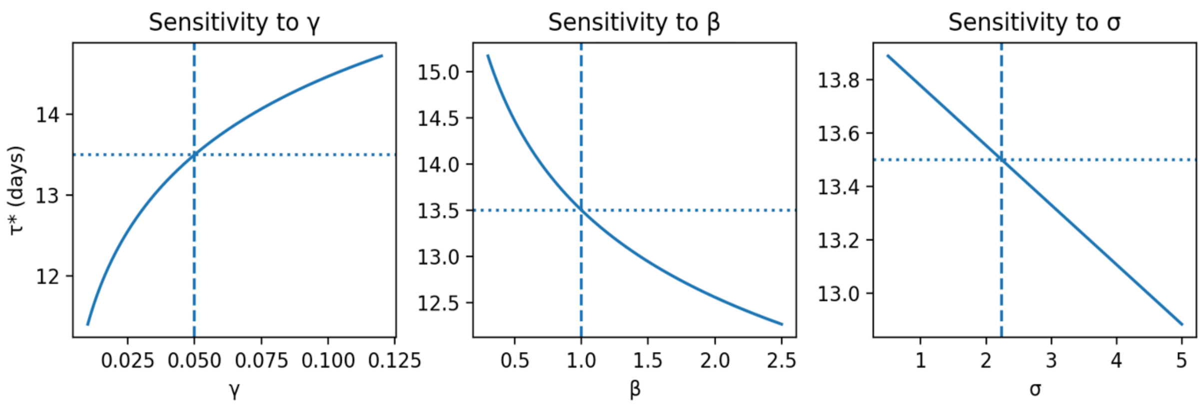

Figure 4. Sensitivity of the optimal delivery time τ* to key model parameters: (a) holding cost γ, (b) shortage loss β, and (c) uncertainty level σ

Conclusion

In this paper, the optimal delivery scheduling time τ^* was determined, at which an optimal balance is achieved between the risk of stockouts and holding costs. It was shown that

$$I(\widetilde{\tau}) = P\{\tilde{\tau} + \Delta\tau \leq \delta_0 + \Delta\delta\}$$

Represents the probability of avoiding a stockout and is a decreasing function; therefore, τ^* is obtained as the unique intersection point of the curves $I(\tilde{\tau})$ and $R(\delta_0)$. A graphical interpretation of the expected total cost was presented, showing that the cost function attains its minimum at τ^*. Taken together, the verification confirms the correctness of the graphical interpretation of the analytical results, the consistency of the numerical calculations with the theoretical conclusions, and the adequacy of the proposed model for analyzing inventory management problems under uncertainty.

An increase in the unit holding cost γshifts the optimum toward later deliveries (τ^* increases), whereas an increase in stockout losses β shifts the optimum toward earlier deliveries (τ^*decreases). Greater uncertainty (higher variability in Δτ and Δδ) makes the curve $I(\tilde{\tau})$ flatter and increases the sensitivity of the outcome to the choice of τ.

Future research directions include extending the proposed model to account for correlations between stochastic deviations in delivery time and inventory depletion time, thereby allowing a more realistic representation of real-world supply chain operating conditions. It also appears promising to develop the model toward multi-item and multi-echelon systems, taking into account network structure and capacity constraints. Another direction is the use of empirical data to estimate distribution parameters and correlation coefficients, as well as calibrating the model for specific industries. Of interest is the study of how alternative distributional assumptions and extreme-disruption scenarios affect the robustness of optimal solutions. In addition, further development may involve integrating the stochastic model into dynamic inventory control problems and into joint planning of supply and production.

Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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